

8.2 Mixed Stackelberg and Cournot Oligopoly

$$X = X_R + X_G = \sum_{i=1}^m x_i + \sum_{i=m+1}^n x_i$$

$$P(X) = b(K - X) + c$$

$$\max_{x_i} \pi_R = P(X)x_i - cx_i$$

$$\frac{d\pi_R}{dx_i} = P(X) + P'(X)x_i - c \stackrel{!}{=} 0$$

$$P(X) + P'(X)x_i = c$$

$$P(X) + P'(X)\frac{1}{m}X_R = c$$

$$b(K - X) + c - b\frac{1}{m}X_R = c$$

$$b(K - X_R - X_G) - b\frac{1}{m}X_R = 0$$

$$K - X_G - \left(1 + \frac{1}{m}\right)X_R = 0$$

$$X_R = \frac{1}{1 + \frac{1}{m}}(K - X_G)$$

$$\max_{X_G} \pi_G = P(X)X_G - cX_G$$

$$\pi_G = P(X)X_G - cX_G$$

$$= (b(K - X) + c)X_G - cX_G$$

$$= b(K - X_R - X_G) + cX_G - cX_G$$

$$= b\left(K - \frac{1}{1 + \frac{1}{m}}(K - X_G) - X_G\right)X_G$$

$$= b\left(\left(1 - \frac{1}{1 + \frac{1}{m}}\right)K - \left(1 - \frac{1}{1 + \frac{1}{m}}\right)X_G\right)X_G$$

$$= bX_G(K - X_G)\frac{1}{1 + m}$$

$$\frac{d\pi_G}{dX_G} = b(K - X_G)\frac{1}{1 + m} - bX_G\frac{1}{1 + m} \stackrel{!}{=} 0$$

$$K - X_G - X_G = 0$$

$$X_G = \frac{1}{2}K$$

Mixed Stackelberg and Cournot Oligopoly (continued)

$$\begin{aligned}X_A &= \frac{1}{\frac{1}{n} + 1}K \\ &= \frac{n}{1+n}K\end{aligned}$$

$$\begin{aligned}X_S &= X_R + X_G \\ &= \frac{1}{\frac{1}{m} + 1} \left(K - \frac{1}{2}K \right) + \frac{1}{2}K \\ &= \frac{m}{1+m} \frac{1}{2}K + \frac{1}{2}K \\ &= \frac{1}{2}K \left(\frac{m}{1+m} + 1 \right) \\ &= \frac{1}{2}K \frac{1+2m}{1+m}\end{aligned}$$

$$X_S > X_A$$

$$\frac{1}{2}K \frac{1+2m}{1+m} > \frac{n}{1+n}K$$

$$(1+2m)(1+n) > 2n(1+m)$$

$$1+2m+n+2mn > 2n+2mn$$

$$1+m > n-m$$