5.2 Spill-over with Side Payments

$$\max_{L_i,S_i,K_i} \quad f(L_i,S_i,K_i) - wL_i - pS_i - rK_i$$

$$f_L = w \qquad f_S = p \qquad f_K = r$$

$$T_i = pS_i$$

$$S_i^* = \frac{1}{n}\sum_{j=1}^{n} S_j \quad \forall i = 1,\dots,n$$

$$\begin{aligned} f(L_i,S_i,K_i) &= f_L L_i + f_S S_i + f_K K_i \\ wL_i + pS_i &= f(L_i,S_i,K_i) - f_K K_i \end{aligned}$$

$$Y_i \equiv wL_i + T_i + r\bar{K} - \sum_{j=1,j\neq i}^{n} \left(Y_j - \bar{Y}_{ij}\right)$$

$$Y_i = f(L_i,S_i,K_i) - r(\bar{K} - K_i) - \sum_{j=1,j\neq i}^{n} \left(Y_j - \bar{Y}_{ij}\right)$$

$$Y_i = f(L_i,S_i,K_i) + r(\bar{K} - K_i) - S_i \sum_{j=1\neq i}^{n} \frac{1}{n}\frac{dY_j}{dS_j^*}$$

$$\max_{S_i,Y_j} \quad U(Y_i,S_i^*)$$

$$\max_{S_i,Y_j} \quad U\left(f(L_i,S_i,K_i) + r(\bar{K} - K_i) - S_i \sum_{j=1\neq i}^{n} \frac{1}{n}\frac{dY_j}{dS_j^*}, \frac{1}{n}\sum_{j=1}^{n} S_j\right)$$

$$\frac{\partial U_i}{\partial S_i} = U_{Y_i}\left(f_S - \sum_{j=1\neq i}^{n} \frac{1}{n}\frac{dY_j}{dS_j^*}\right) + \frac{1}{n}U_{S_i} \stackrel{!}{=} 0$$

$$f_S - \sum_{j=1\neq i}^{n} \frac{1}{n}\frac{dY_j}{dS_j^*} = \frac{1}{n}\left(-\frac{U_{S_i}}{U_{Y_i}}\right)$$

$$f_S - \sum_{j=1\neq i}^{n} \frac{1}{n}\frac{dY_j}{dS_j^*} = \frac{1}{n}\frac{dY_i}{dS_i^*}$$

$$f_S = \sum_{j=1}^{n} \frac{1}{n}\frac{dY_j}{dS_j^*}$$

$$f_S = \frac{dY_j}{dS_j^*}$$